SPECIAL focus TUTORIAL

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Living with UNCERTAINTY

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UNDERSTANDING AND QUANTIFYING UNCERTAINTY IS KEY TO ACCURATE AND COST-EFFECTIVE TESTING.

Webster's Collegiate Dictionary defines uncertainty as "the quality or state of being indefinite, indeterminate, or not reliable: doubt." Can you imagine a customer asking your metrology or test staff if they doubt their measurements? Unfortunately, the answer must always be yes...to an extent. The extent is what we define as the measurement uncertainty.

A perfect measurement would obtain the true value of a quantity, which is the value consistent with the definition of a given quantity.[1] True values are, by nature, indeterminate because a perfect measurement cannot be performed. In fact, says the International Organization for Standardization (ISO), it is impossible to fully describe the measurand (the quantity to be measured) without an infinite amount of information. In other words, the final corrected result of a measurement is, at best, an estimate of the true value of the quantity that someone intended to measure. The measurement uncertainty is a parameter that characterizes the dispersion of the values that could reasonably be attributed to the measurand.

Confusion exists, even among scientists and engineers, as to the exact meaning of terms like accuracy, precision, and error. The accuracy of measurement is the closeness of agreement between the result of a measurement and a true value of the measurand. The error of measurement is the result of a measurement minus a true value of the measurand. As stated above, the true value of a quantity cannot be known; therefore, accuracy and error cannot be quantified for a measurement result. Precision is the closeness of agreement for independent measurements obtained under stipulated conditions.[2] Repeatability is the closeness of the agreement between the results of successive measurements carried out under the same conditions of measurement during a short period of time. Reproducibility is the closeness of the agreement between the results of measurements carried out under changed conditions of measurement. Precision may be stated as "precision under repeatability conditions" or "precision under reproducibility conditions." These terms (accuracy, error, and precision) should not be used to specify the uncertainty of measurements.

The Cost of Uncertainty

Uncertainty is significant because it can affect the cost of production. A particular quality of a product must fall within a given tolerance limit to meet a specification. If the product quality does not fall within the tolerance limit, it is rejected. We make this judgment by measurement, but to do so effectively, we must define how close the measured results can be to the tolerance. The tolerance interval for the measured value must be smaller than the tolerance of the product (see figure). Here, the measurement uncertainty plays an important role. The smaller the uncertainty, the closer the measurement results can approach the tolerance limit without being rejected.

Properly understood, uncertainty can also provide a useful tool in the planning phase of production. By completing an uncertainty budget (a list of the uncertainty components and their respective uncertainty contributions), a manager can immediately determine which sources of uncertainty contribute most and reduce the largest sources of uncertainty by spending money wisely.

Uncertainty of measurement is becoming increasingly important as manufacturers implement quality systems on the production floor. Quality standards like ISO 9000 and ISO/IEC 17025 require traceability of all measurements and calibrations performed to products. The ISO defines traceability as the property of the result of a measurement or the value of a standard that allows that standard to be related to stated references (generally national or international standards) through an unbroken chain of comparisons that possess stated uncertainties. This means that there should be a chain of calibration certificates between the instruments used by the manufacturers and the national standards, and all certificates involved must have uncertainty statements.

NIST-traceable is a commonly used term, but it does not, in and of itself, guarantee the lowest uncertainty. If the calibration chain is too long or the intermediate calibrations are not of high quality, the uncertainty may become significantly large, much larger than that of the NIST scale. As long as an unbroken chain of comparisons with stated uncertainties is made, however, the measurement is considered traceable to NIST. It is important to understand that NIST is not a regulatory body. It is the customer's job to request and verify the calibration chain. A reputable laboratory will provide this information leading back to a NIST calibration report.

Determination of Uncertainty

To ensure that calibration certificates are properly interpreted by all calibration laboratories worldwide, it is critical that we state uncertainty values in a uniform manner. Over the years, the community has taken many different approaches to evaluating and expressing measurement uncertainty. In 1977, the International Committee for Weights and Measures (Comité International des Poids et Measures, CIPM) asked the International Bureau of Weights and Measures (Bureau International des Poids et Measures, BIPM) to collaborate with the various national metrology institutes to propose a solution to this problem. The result was Recommendation INC-1 (1980), *Expression of Experimental Uncertainties.* Recommendation INC-1 (1980) provided only a brief outline of requirements, however, so CIPM asked the ISO to develop a comprehensive guide based on INC-1.

In 1993, the final work was published as the *Guide to the Expression of Uncertainty in Measurement* (GUM), which was ultimately printed in 1995.[3] Seven international organizations supported the development of this document, which has become the definitive authority on uncertainty evaluation worldwide. Recently, a new international organization, the Joint Committee for Guides in Metrology (JCGM), was formed to assume responsibility for maintenance and revision of the GUM.[4]

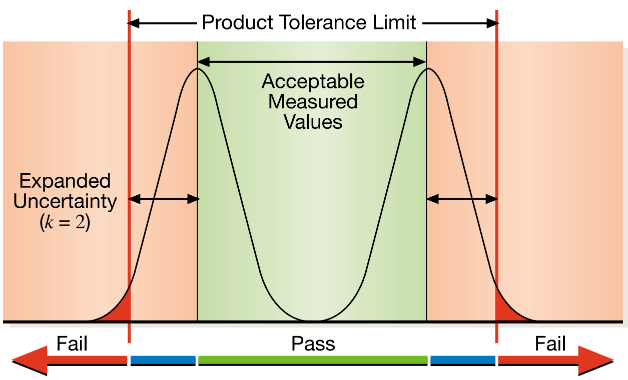
The GUM describes eight steps for the determination of measurement uncertainty. Step One involves writing a measurement equation for the measurand. A measurement equation is expressed as a mathematical function $Y = f(X_1,...,X_n)$ that represents the process that is used to determine the result of a measurement and its associated standard uncertainty.[5] The quantities X_i are not only parameters for physical laws, but can include corrections and other quantities that account for sources of variability. Often, the corrections have a magnitude of one and do not affect the value of the result, but possess inherent uncertainty. A test method, for example, may state the temperature must be 25.0°C. If the temperature at the time of measurement is 25.0°C with an uncertainty of 0.5°C, the uncertainty of 0.5°C contributes to the overall uncertainty of the measurement result. In many situations, defining all of the parameters that indirectly affect a measurement is the most difficult step. Step Two determines all of the input quantities x_i, the estimated values of X_i, based on statistical analysis of a series of measurements or by other means.

Step Three determines the standard uncertainty, $u(x_i)$, associated with each estimate x_i. The GUM categorizes components of uncertainty by the methods used to evaluate them. Type A evaluation is based on statistical analysis of a series of observations. An uncertainty component obtained by a Type A evaluation is a statistically estimated standard deviation s_i. The standard uncertainty for a Type A evaluation is $u(x_i) = s_i$. Type B evaluation is based on means other than statistical analysis; that is, scientific judgment using all relevant information, including previous measurement data, previous experience, manufacturers' specifications, and data-in-calibration reports. Typically, Type B evaluations are based on probability distributions, such as a uniform distribution on some interval [a, b]. Specification of a uniform distribution requires information on the upper and lower bounds only. We can determine the mean quantity and standard deviation of a uniform distribution and other probability distributions using moment-generating functions. A uniform distribution, for example, has a mean μ and a standard deviation σ of

$$\mu = \frac{a+b}{2} \qquad \sigma = \frac{(b-a)}{2\sqrt{3}}$$

In this case, the estimate equals the mean ($x_i = \mu$) and the standard uncertainty equals the standard deviation ($u(x_i) = \sigma$). For a further discussion of common probability distributions for Type B evaluation, see the GUM.

Step Four evaluates the correlation coefficient for all pairs of estimates. Two quantities are considered correlated when they are affected by a common quantity. When we measure the luminous intensity of a lamp with a photometer, for example, the lamp current and color temperature are quantities of the measurement equation. The current and color temperature are correlated because as the current increases, so does the color temperature. Another example of correlation is measuring a length standard with four NIST-calibrated meter sticks. If we ignore correlation, the calibrated length standard would appear to have an uncertainty half that of the meter sticks. In reality, the uncertainty would barely decrease—the four calibrated instruments have a strong correlation because their calibration is based on the same unit realization. The correlation coefficient, $r(x_i,x_j)$, is the covariance divided by the standard deviation of the two quantities. The correlation coefficient has a range of -1 to +1, and it can reduce or enlarge the combined standard uncertainty. The value of the correlation coefficient can be found by additional statistical analysis as presented in the GUM.



For a product to pass a test, the measured value must exceed the tolerance and the expanded uncertainty (green area). The expanded uncertainty defines a confidence interval. Due to the confidence interval, products that are not acceptable will be taken as acceptable (solid red area) or indeterminate (blue).

Step Five involves calculating the result of measurement y for Y from the measurement equation using the estimates x_i for the input quantities X_i; thus, $y = f(x_1,\ldots,x_n)$. Step Six determines the combined standard uncertainty $u_c(y)$. The combined standard uncertainty consists of the estimated standard deviation for the measurand, which is the positive square root of the estimated variance $u_c^2(y)$. The variance is approximated by a first-order Taylor series of the measurement equation about y. The Taylor series approximation gives the law of propagation of uncertainty:

$$u_c^2(y) = \sum_{i=1}^{N}\left(\frac{\partial Y}{\partial X_i}\bigg|_{x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial Y}{\partial X_i}\bigg|_{x_i}\frac{\partial Y}{\partial X_j}\bigg|_{x_j} u(x_i)u(x_j)r(x_i,x_j)$$

The partial derivative of Y with respect to X_i is called the sensitivity coefficient, which can be determined experimentally if the exact functional relation cannot be expressed in a closed form.

Step Seven produces the expanded uncertainty U, which defines an interval $[y \pm U]$ about the measurement result that encompasses a large fraction of the distribution of values that could reasonably be attributed to the measurand. Standard uncertainty is converted to expanded uncertainty by multiplying a coverage factor k. This factor is selected on the basis of the level of confidence or coverage probability required. The level of confidence depends on the distribution for Y represented by y and $u_c(y)$ and the coverage factor k. If the distribution is normal and $k = 2$, the coverage probability is 95%. If the distribution is uniform and $k = 2$, the coverage probability is 100%. The minimum coverage probability with $k = 2$ for any distribution that has an expected value of y and standard deviation of $u_c(y)$ is 75%. NIST policy on expression of uncertainty states the value of k to be used in the NIST calibration services for calculating U is $k = 2$. Finally, Step Eight involves reporting the result of measurement y along with its combined standard uncertainty $u_c(y)$ or expanded uncertainty U and describing how y, $u_c(y)$, and U were obtained.

Traceability to a standard is not sufficient to guarantee an effective measurement. To achieve meaningful results without incurring undue cost, it is important to understand uncertainty, know your calibration chain, and plan an efficient testing process. **oe**

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References

1. International Vocabulary of Basic and General Terms in Metrology, International Organization for Standardization (1993).

2. B. Taylor and C. Kuyatt, NIST Technical Note 1297 (1994). See physics.nist.gov/Document/tn1297.pdf.

3. Guide to the Expression of Uncertainty in Measurement, International Organization for Standardization (1995).

4. See www.bipm.org.

5. R. Kacker and A. Jones, Metrologia, 40, p. 235 (2003).