Engineering Geodesy I

Exercise 3:

Monitoring of a Bridge

Determination of the Eigenfrequencies

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Model Analyses of the Bridge

theoretical Eigenfrequencies:

EW-Nr.	w^2	w	Periode	Frequenz	Feld	Richtung	Nummer der Eigen-schwingung	Bezeichner
	[(rad/s)2]	[rad/s]	[s]	[s-1]				
1	35.9	5.99	1.049	0.95	2	Vertikal	1	2V1
2	48.2	6.95	0.905	1.11	2	Torsion	1	2T1
3	51.5	7.18	0.876	1.14	2	Vertikal	2	2V2
4	88.5	9.41	0.668	1.50	2	Torsion	2	2T2
5	110.9	10.53	0.597	1.68	2	Horizontal quer	1	2S1
6	121.3	11.01	0.571	1.75	2	Vertikal	3	2V3
7	175.3	13.24	0.475	2.11	2	Torsion	3	2T3
8	213.8	14.62	0.430	2.33	2	Vertikal	4	2V4
9	283.3	16.83	0.373	2.68	2	Torsion	4	2T4

Preparation of Data for Analyses

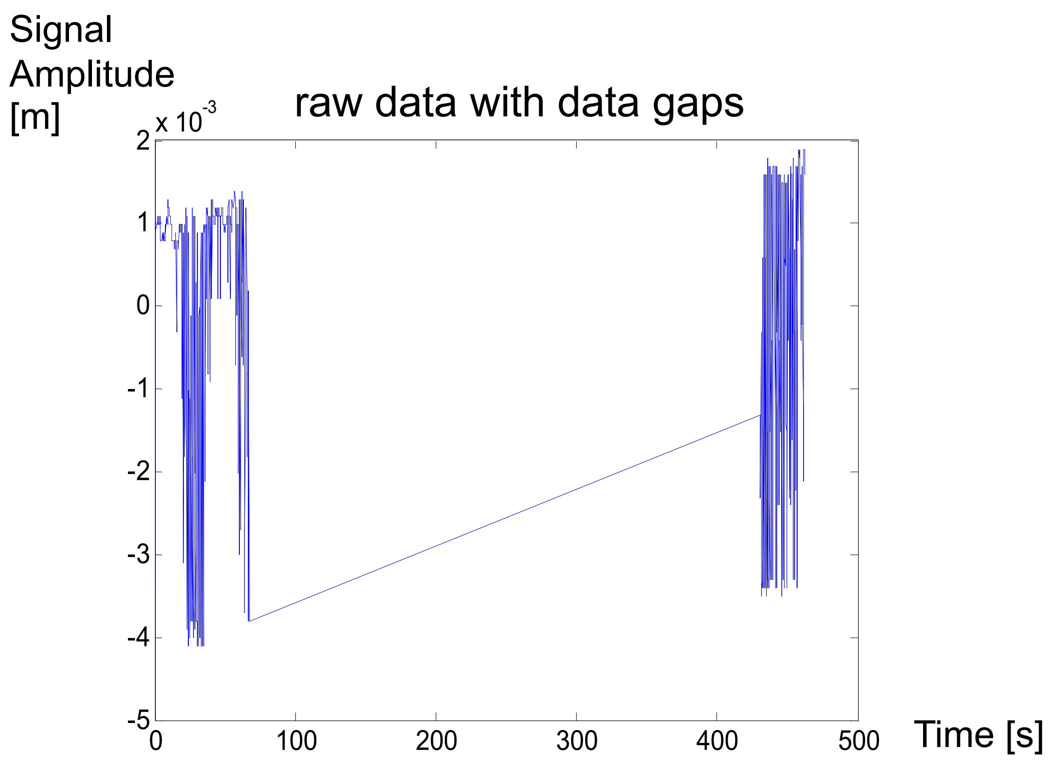
1. Data should be brought in a two-column form (for each dimension separately:

measurement time	measurement reading
t_1	y_1
t_2	y_2
...	...
t_n	y_n

convert time into decimal [s]:
10/14/2010 11:09:58

separator [:]
h*3600 + min*60 + s

raw data with data gaps



Preparation of Data

2. Elimination of outliers

- use of threshold
- visual detection of single jumps
- outlier detection using a filter (e.g. running average)

3. Cutting out relevant interval

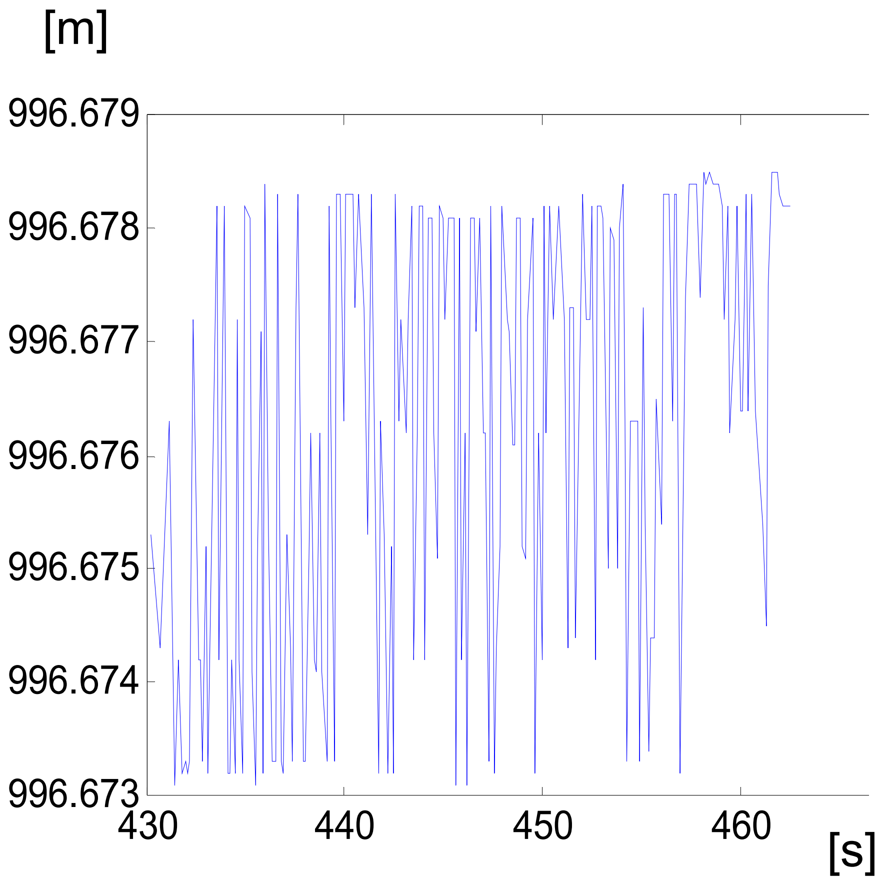
Remaining interval should not have

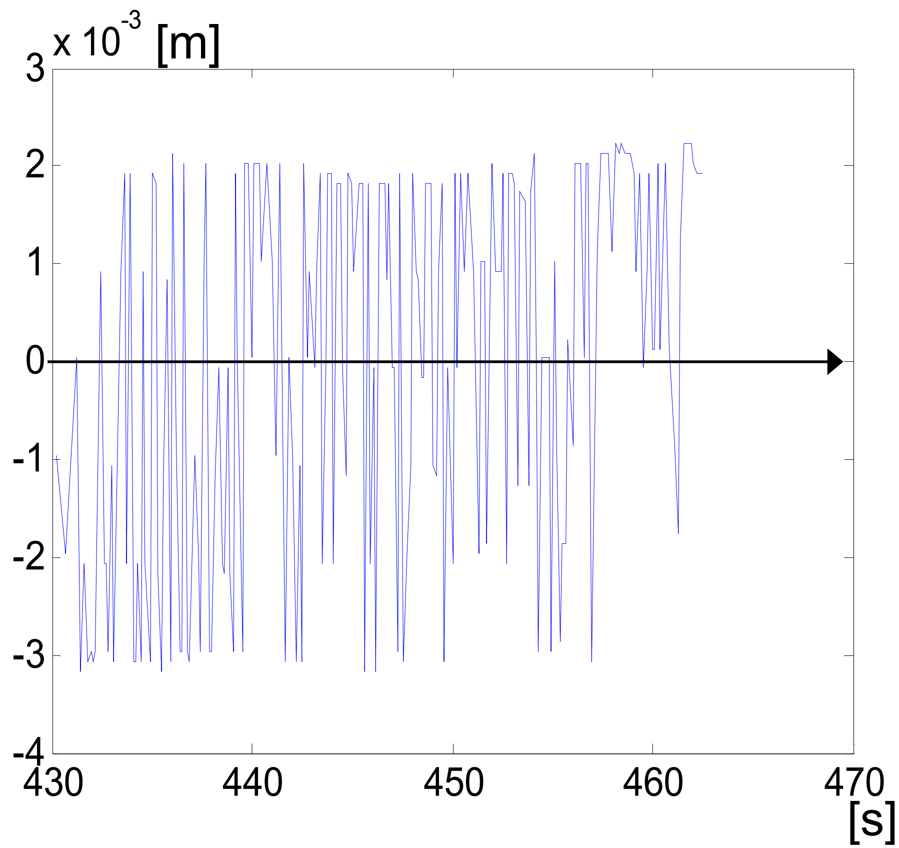
- jumps
- systematic bias & drift
- no data gaps (if FFT is used)

Preparation of Data

4 . Take out bias and trend

Model: $y = a\,t + c$ (a = trend or drift, t = time, c = constant bias or offset)





Determination of a Harmonic Sine Function in a Time Series

Model function:

$$\mathbf{y} = f(\mathbf{t}) = a\sin(2\pi\mathbf{t}f + \varphi)$$

time	value
t_1	y_1
t_2	y_2
...	...
t_n	y_n

<u>unknown parameters:</u>

amplitude	a
frequency	f
initial phase	φ

abbreviation: $x_i = 2\pi t_i$

a and φ are replaced using addition theorem for sine

$$f(\mathbf{x}) = a\sin(f\mathbf{x} + \varphi)$$

$$f(\mathbf{x}) = a\sin(f\mathbf{x} + \varphi) = a\sin(f\mathbf{x})\cos(\varphi) + a\cos(f\mathbf{x})\sin(\varphi)$$

$$A = a\cdot\cos(\varphi) \qquad B = a\cdot\sin(\varphi)$$

$$f(\mathbf{x}) = A\sin(f\mathbf{x}) + B\cos(f\mathbf{x})$$

A, B and f are the new parameters now

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Determination of a Harmonic Sine Function in a Time Series

FFT (Fast Fourier Transform):
Excel: Tools → Add-ins → Analysis Tool Pack, choose Fourier Analysis

requirements:
- equidistant time intervals
- 2^n data pairs (or pad end with zeros up to the next 2^n-th data pair)
- uses only one column

result: a + bi (imaginary numbers)
amplitude = WURZEL((IMREALTEIL(Result))^2+(IMAGINÄRTEIL(Result))^2)
→ spectrum ([1..*n*/2] cycles per dataset)

Matlab:
y = fft(X)
returns the Discrete Fourier Transform (DFT) of vector X,
performs automatic zero padding
correct amplitudes: **abs(fft(X))/n*2** (plot only relevant part of the spectrum)

correct frequencies: $f_i = (y_i - 1) / T$,
where
y_i = i-th value in fft vector
T = total measurement duration

Periodogram Function

Purpose: best fit of a sine-curve to the data (given any frequency *f*)

$e_i = A\sin(fx_i) + B\cos(fx_i) - y_i$ observation equation abbreviation: $x_i = 2\pi t_i$

$F = \sum_{i=1}^{n} \left(A\sin(fx_i) + B\cos(fx_i) - y_i \right)^2 = \text{min.}$ objective function (3 parameters)

normal equations in matrix form $\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} S & G \\ G & C \end{bmatrix}^{-1} \begin{bmatrix} a \\ b \end{bmatrix}$

$S = \sum_{i=1}^{n} \sin(fx_i)\sin(fx_i)$ $C = \sum_{i=1}^{n} \cos(fx_i)\cos(fx_i)$

$G = \sum_{i=1}^{n} \sin(fx_i)\cos(fx_i)$

$a = \sum_{i=1}^{n} y_i \sin(fx_i)$ $b = \sum_{i=1}^{n} y_i \cos(fx_i)$

the solution is: $A = \dfrac{C\cdot a - G\cdot b}{S\cdot C - G^2}$ $B = \dfrac{S\cdot b - G\cdot a}{S\cdot C - G^2}$

Periodogram reads: $P(f) = \sqrt{A^2 + B^2}$

Periodogram *P* depends only on the parameter *f* (frequency)

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Periodogram

m-file haupt.m:
on geodesy engineering website (two m-files: `haupt.m` and `calc_mag.m`)
parameters:

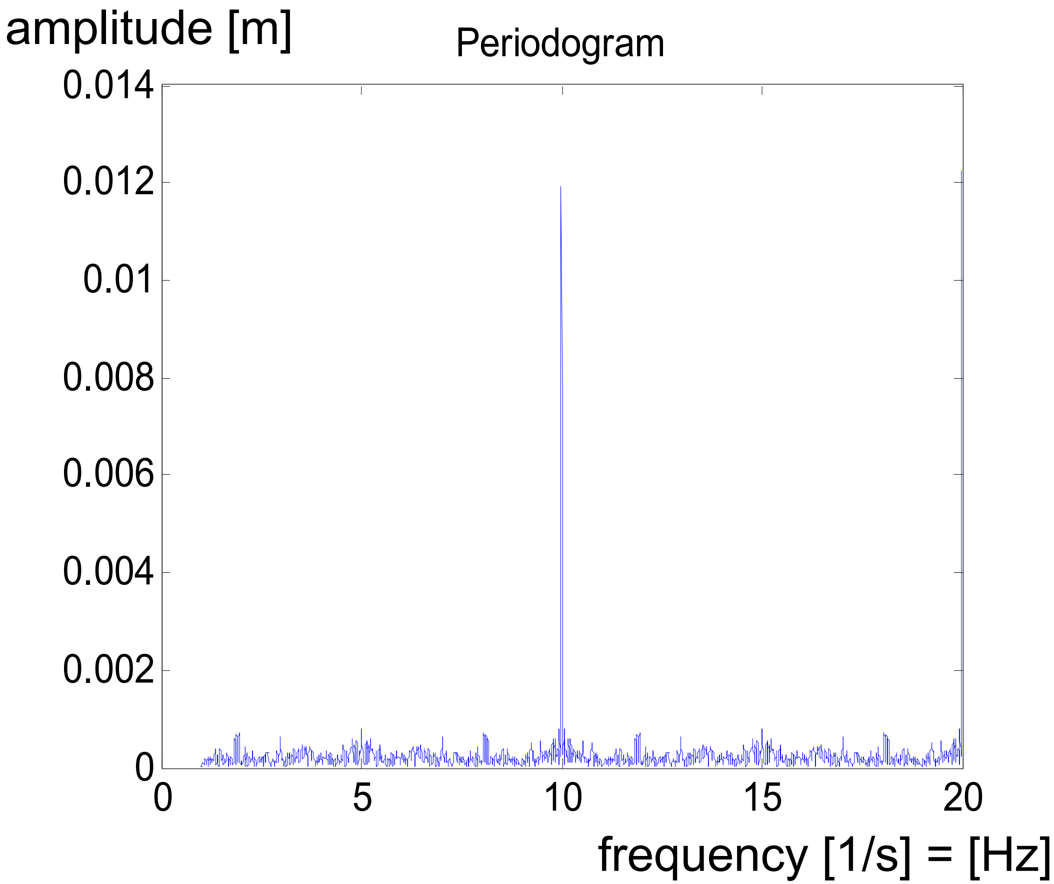
```
input:  a file with a data series (simple ascii; two columns)
fist column: time, second column: data values
```

```
load tps.beo;
torg = tps(:,1);
yorg = tps(:,2);

Variable “detail” is the resolution. Use 0.1 to sample the
periodogram 10 times finer than Fourier (=the data series resolution)
detail = 0.1;

% choose frequency interval for periodogram
fmin = 1;   % [1/s]
fmax = 20;  % [1/s]

m = 1;  %number of unknown frequencies that are to be found in the
data
```

Periodogram

result for
f_{min} = 1 Hz
f_{max} = 20 Hz

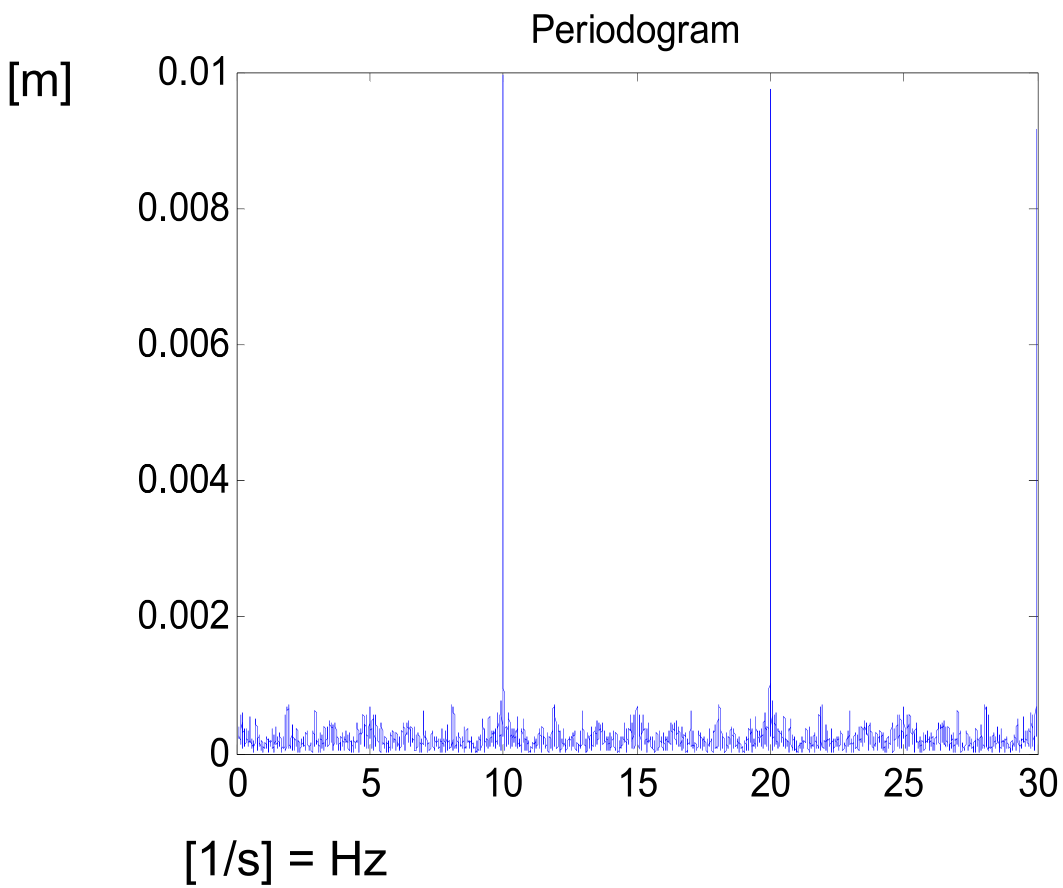
n = 203
equidistant, Δt = 0.1
duration T = 20.3 s



Periodogram

result for
f_{min} = 0.1
f_{max} = 30

n = 203
equidistant, Δt = 0.1 s

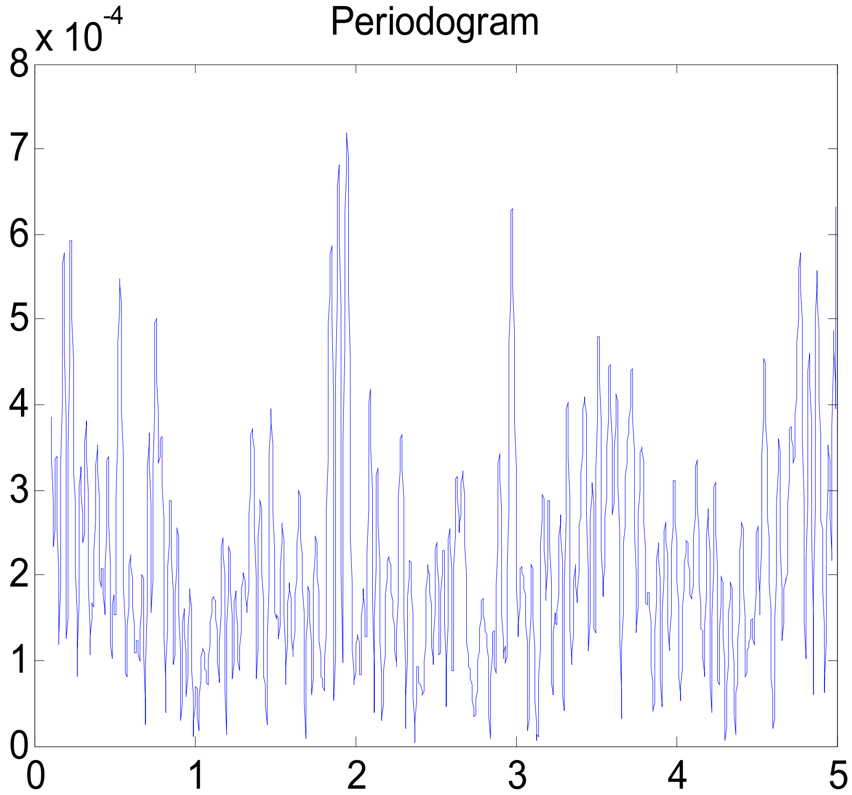


Periodogram

result for
f_{min} = 0.1
f_{max} = 5

n = 203
equidistant, Δt = 0.1 s

[m]



[1/s] = Hz

Result: frequency(1) = 1.942 [1/s] amplitude(1) = 0.0007 [m]